Instability of Optimal Stochastic Control Systems under Parameter Variations

JASON L. SPEYER*
Analytical Mechanics Associates Inc., Cambridge, Mass.

The deterioration of a linear optimal stochastic control scheme, designed under the assumptions of the certainty-equivalence principle (the optimal filter and controller, determined independently, combine to give a totally optimal system), is investigated when the parameters of the actual system do not coincide with the design values. This linear suboptimal stochastic system is described by a covariance matrix composed of covariances of the estimates of the state variables, the errors in the estimates of the state variables, and the correlation between these errors and the estimates. In particular, this paper is concerned with the covariance matrix resulting from a single state dynamical system and a scalar linear measurement function of both the state variable and the control variable (e.g., accelerometer measurements). A modeling error in the control variable coefficient of the measurement function may induce instability in the stochastic system with either unstable or stable dynamics. Furthermore, the absolute magnitude of the error in the control variable coefficient directly influences system stability, not the relative error. Thus, relatively small errors compared to the design value of this coefficient may be quite important.

I. Introduction

ALTHOUGH there are many studies on divergence of optimal filters, little attention has been given to the effect of modeling inaccuracies on optimal linear stochastic control systems. Here we extend Fitzgerald's[1] investigation of Kalman filter divergence to optimal linear stochastic control systems. These systems are designed under the certainty equivalence principle[2] which states that if the expected value of a quadratic function of the state and the control variables is to be minimized subject to linear dynamics, the optimal system is composed of an optimal filter in cascade with an optimal controller. This separation is possible because the estimate in the state is uncorrelated with the error in this estimate.

If the parameters in the assumed model of the dynamics or the measurement device deviate from the parameters of the actual system, the estimate and the error in the estimate become correlated. The behavior of the system because of the gains based on an inaccurate model is studied by considering the coupled matrix covariance equation composed of the covariances of the error in the estimate, the estimate, and the estimate with its error.

Some of the characteristics of this linear matrix equation are studied through a scalar linear dynamic equation. The errors in system parameters enter into the 2×2 covariance equation in a dimensionless form allowing the following general results to be obtained: 1) The stochastic control system may be unstable when the nonoptimal filter and deterministic control systems individually are stable. 2) Instability occurs only when the error in the parameter exceeds a finite threshold value. 3) If the measurement is a linear function of the control variable as well as the state (e.g., accelerometer measurements) and there are errors in the coefficient of the control, then instability of the total system may occur for both stable and unstable dynamical systems. 4) The filter or control gains are not functions of the coefficient of the control variable in the measurement function. Consequently, the absolute magnitude of the error in this coefficient influences stability. However, for the other system parameters the relative magnitude of the error in the design value influences system stability. 5) The only case for which errors in the process noise covariances cause instability is when the system is neutrally stable and the system is designed assuming zero process noise.

Presented as Paper 70-36 at the AIAA 8th Aerospace Sciences Meeting, New York, January 19–21, 1970; submitted March 3, 1970; revision received August 17, 1970.

* Senior Analyst; presently at Charles Stark Draper Laboratory, Massachusetts Institute of Technology, Cambridge, Mass. Member AIAA.

II. System Equations, Estimator, and Controller

The dynamic equation and the measurement device are assumed to be linear although time varying

$$\dot{\mathbf{x}} = F(t)\mathbf{x} + G(t)\mathbf{u} + \mathbf{v} \tag{1}$$

$$\mathbf{z} = H(t)\mathbf{x} + M(t)\mathbf{u} + \mathbf{w} \tag{2}$$

where $\mathbf{x}$ in an n vector of state variables, $\mathbf{u}$ is a p vector of control variables, $\mathbf{z}$ is a q vector of measurements, $\mathbf{v}$ is an n vector of white Gaussian process noise and $\mathbf{w}$ is a q vector of white Gaussian measurement noise. The sufficient statistics of $\mathbf{v}$ and $\mathbf{w}$ are given by a zero mean and covariance

$$E\{\mathbf{v}(t)\mathbf{v}^{\mathrm{T}}(\tau)\} = Q(t)\delta(t - \tau) \tag{3}$$

$$E\{\mathbf{w}(t)\mathbf{w}^{\mathrm{T}}(\tau)\} = R(t)\delta(t - \tau) \tag{4}$$

where $E\{\ \}$ means expected value of $\{\ \}$. The matrices $F(t)$, $G(t)$, $H(t)$, $M(t)$, $Q(t)$, $R(t)$, are assumed in this analysis to be the actual system parameters. These same variables subscripted by c are the parameters used in designing the filter and controller. The previous equations could represent the linearization about some nominal path described by a set of nonlinear differential equations. In this application, the parameters used in designing the estimator and controller are first-order partial derivatives evaluated on the nominal path. When the path deviates from the nominal path, the parameters deviate from the design values.

The estimator and controller theoretically can be determined separately (based upon the certainty-equivalence principle) if the expected value of a quadratic form of the state and control variables is to be minimized, subject to the differential constraint Eq. (1), by a choice of the estimator gain $K(t)$ and control $\mathbf{u}(t)$. The control $\mathbf{u}(t)$ is found to be a linear combination of the estimates in the states $\hat{\mathbf{x}}$ as

$$\mathbf{u} = -\Lambda(t)\hat{\mathbf{x}} \tag{5}$$

where $\Lambda(t)$ is a $p \times n$ matrix. The estimate of the state $\hat{\mathbf{x}}$ is determined by solving the linear equation

$$\dot{\hat{\mathbf{x}}} = (F_c - G_c\Lambda)\hat{\mathbf{x}} + K(t)(\mathbf{z} - H_c\hat{\mathbf{x}} + M_c\Lambda\hat{\mathbf{x}});\ \hat{\mathbf{x}}(t_o) = 0 \tag{6}$$

where the controller of Eq. (5) has been substituted into Eq. (6). The optimal design value of the filter gain matrix is given in Ref. 2 as

$$K = P_cH_c^TR_c^{-1} \tag{7}$$

where the calculated covariance of the error in the estimate P_c is propagated forward in time by the Riccati equation

$$\dot{P}_c = F_cP_c + P_cF_c^T - P_cH_c^TR_c^{-1}H_cP_c + Q_c;\ P_c(t_o) = P_{co} \tag{8}$$

The optimal controller gain matrix $\Lambda(t)$ which minimizes the quadratic performance index

$$J = E\left\{\mathbf{x}^TC\mathbf{x}|_{t=tf} + \int_{t_o}^{t_f}[\mathbf{x}^TA(t)\mathbf{x} + \hat{\mathbf{x}}^T\Lambda^TB(t)\Lambda\hat{\mathbf{x}}]dt\right\} \tag{9}$$

subject to Eq. (1) is given in Ref. 2 as

$$\Lambda(t) = B^{-1}G_c^TU \tag{10}$$

where U is propagated backwards in time by the Ricatti equation

$$\dot{U} = -F_c^TU - UF_c + UG_cB^{-1}G_c^TU - A;\ U(t_f) = C \tag{11}$$

The block diagram of the stochastic control system is given in Fig. 1. This system is optimal only if the design parameters (subscripted with a c) coincide with the system parameters.

III. Statistical Analysis of System with Model Inaccuracies

If there are errors in the system parameters, the estimate and the error in the estimate are correlated. To describe the behavior of the covariance of this coupled system, a linear $2n \times 2n$ matrix differential equation is now determined.

The dynamic equation, using Eq. (5) in Eq. (1), is

$$\dot{\mathbf{x}} = F\mathbf{x} - G\Lambda\hat{\mathbf{x}} + \mathbf{v} \tag{12}$$

The estimator equation of Eq. (6) rewritten to emphasize the modeling errors is

$$\dot{\hat{\mathbf{x}}} = (F - G\Lambda + D)\hat{\mathbf{x}} - KH\mathbf{e} + K\mathbf{w} \tag{13}$$

where all the modeling errors are included in D

$$D = -[(F - F_c) - (G - G_c)\Lambda] + K[(H - H_c) - (M - M_c)\Lambda] \tag{14}$$

The error in the estimate, defined as

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x} \tag{15}$$

is propagated by

$$\dot{\mathbf{e}} = (F - KH)\mathbf{e} + D\hat{\mathbf{x}} + K\mathbf{w} - \mathbf{v} \tag{16}$$

Since $\mathbf{e}$ and $\mathbf{x}$ are coupled, the statistical properties of the system are found by the first two moments of the composite vector $[\mathbf{e}^T, \hat{\mathbf{x}}^T]$. The first moment is

$$E\left\{\begin{matrix}\mathbf{e}\\ \hat{\mathbf{x}}\end{matrix}\right\} = 0 \tag{17}$$

The covariance matrix is

$$E\left\{\begin{bmatrix}\mathbf{e}\\ \hat{\mathbf{x}}\end{bmatrix}[\mathbf{e}^T\mathbf{x}^T]\right\} = \begin{bmatrix}P & S\\ S^T & \hat{X}\end{bmatrix} = Y \tag{18}$$

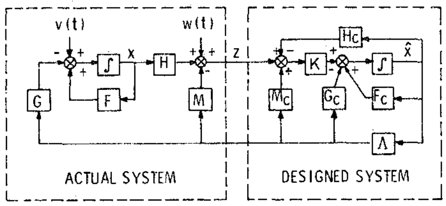

Fig. 1 Stochastic control system, optimal if ($)_c$ = ( ).

If there are no parameter errors, then the estimate and the error in the estimate are uncorrelated as

$$S = E\{\mathbf{e}\hat{\mathbf{x}}^T\} = 0 \tag{19}$$

The covariance matrix Y is propagated by the $2n \times 2n$ matrix differential equation

$$\dot{Y} = \Gamma Y + Y\Gamma^T + \Xi \tag{20}$$

where

$$\Gamma = \begin{bmatrix}(F - KH) & D\\ -KH & (F - G\Lambda + D)\end{bmatrix} \tag{21}$$

$$\Xi = \begin{bmatrix}KRK^T + Q & KRK^T\\ KRK^T & KRK^T\end{bmatrix} \tag{22}$$

The effect of modeling errors on the steady-state performance of the system is characterized by the eigenvalues of Eq. (21) when Γ is time invariant.† For large enough parameter variations the real part of the eigenvalues of Γ can become positive causing instability in Y. Furthermore, due to the coupling introduced by D, both the estimate and error in the estimate become unbounded. However, even if the steady-state performance of the system is tolerable, the transient response may be unacceptable.

IV. Observation about Errors in System Parameters

All the errors in the system parameters are contained in D of Eq. (14) except for the errors in R and Q. Modeling errors in R and Q have a secondary effect on the eigenvalues of Γ and do not usually induce system instability (see Sec. 5.4). Furthermore, the parameter errors entering D are additive, allowing small errors in the individual parameters to possibly result in a large effective error. Notice that errors in G, H, and M are amplified by the system gains. In fact errors in M are multiplied by both filter and control gains further amplifying system degeneration. The matrix M is unique in that it does not enter into the design of either the filter gain or the controller gain. Examples of a measurement device which involve M are the accelerometer and inertial platform which, for example, may measure thrust and aerodynamic forces.

V. Scalar Problem

The effects of modeling error on system behavior are studied both analytically and by use of a computer simulation through a scalar time invariant dynamic system and measurement device. The analysis of the previous sections applies to this scalar problem where equivalent lower case letters are used to indicate scalars. The Γ of Eq. (21) is now a 2×2 matrix of the form

$$\mathbf{\Gamma} = \begin{bmatrix}f - kh, & d\\ -kh, & f - g\lambda + d\end{bmatrix} \tag{23}$$

† Note that there exist time varying linear systems with positive eigenvalues that are stable.[3]

V.I. Steady-State Gains

The eigenvalues of Γ in Eq. (23) are of interest when the system gains reach a steady-state condition. The controller gain is a constant if the control interval is assumed infinite allowing U of Eq. (11) to attain steady state when integrated backwards in time. When $\dot{P}_c = 0$ in Eq. (8) and $\dot{U} = 0$ in Eq. (11) the resulting steady state optimal filter and control gains are

$$k = [1 \pm \zeta]f_c/h_c \tag{24}$$

$$\lambda = [1 \pm \eta]f_c/g_c \tag{25}$$

where

$$\zeta = [1 + (q_c/r_c)(h_c/f_c)^2]^{1/2}; \quad 1 < \zeta < \infty \tag{26}$$

$$\eta = [1 + (a/b)(g_c/f_c)^2]^{1/2}; \quad 1 < \eta < \infty \tag{27}$$

and if f_c is positive then the + sign is used; if f_c is negative use − sign.

Using the constant gains of Eqs. (24) and (25) in Eq. (23)

$$\Gamma = -f_c\begin{bmatrix} \{\pm\zeta - \Delta f + \Delta h(1 \pm \zeta)\}, & \{\Delta f - \Delta g(1 \pm \eta) - \Delta h(1 \pm \zeta) + (\Delta m f_c/g_c h_c)(1 \pm \zeta)(1 \pm \eta)\} \\ \{(1 + \Delta h)(1 \pm \zeta)\}, & \{\pm\eta - \Delta h(1 \pm \zeta) + (\Delta m f_c/g_c h_c)(1 \pm \zeta)(1 \pm \eta)\} \end{bmatrix} \tag{28}$$

where

$$\Delta f = (f - f_c)/f_c,\ \Delta g = (g - g_c)/g_c,\ \Delta h = (h - h_c)/h_c,\ \Delta m = m - m_c \tag{29}$$

Note from Eq. (29) that in this scalar problem the *relative* errors of f, g, h enter directly into Eq. (28). However, the *magnitude* of the error in m, Δm, directly affects the system performance. A small relative error in m, $\Delta m/m_c$, may cause serious system degeneration.

The question of instability is determined when the real part of the eigenvalues of Eq. (28) first become positive. In the following sections, conditions for instability are determined by examining separately each parameter error.

V.II. Instability due to Variations in f, g, and h

Let us first consider errors in g. If $f_c > 0$ then from Eq. (28)

$$-\frac{\Gamma}{f_c} = \begin{bmatrix} \zeta & -\Delta g(1 + \eta) \\ 1 + \zeta & \eta \end{bmatrix} \tag{30}$$

If ν is an eigenvalue of Eq. (30) then the characteristic equation is

$$\nu^2 - (\zeta + \eta)\nu + \zeta\eta + \Delta g(1 + \zeta + \eta + \zeta\eta) = 0 \tag{31}$$

A zero eigenvalue is obtained when the variation of g, Δg, reaches a critical variation Δg_{cr}

$$\Delta g_{cr} = -[\zeta/(1 + \zeta)][\eta/(1 + \eta)]; \quad \Delta g_{cr} \leq -0.25 \text{ for } f_c > 0 \tag{32}$$

Since ζ and η have ranges given in Eqs. (26) and (27), the system will always be stable if $\Delta g > -0.25$ regardless of the values of the system parameters. However, for values of Δg between −0.25 and −1 instability does depend upon the system parameters and will occur when Δg_{cr} is reached. The system is always unstable if $\Delta g < -1$ because g is of opposite sign from g_c causing the system to have positive feedback. If $f_c < 0$, instability can only occur if $\Delta g < -1$.

If a similar analysis is applied to Δh, identical results are obtained as found for Δg [i.e., the critical value of Δh, Δh_{cr} is exactly the same as Δg_{cr} given in Eq. (32)]. Note the duality between h and g.

If only variations in f occur then the system diverges when Δf reaches a critical value Δf_{cr}

$$\Delta f_{cr} = \zeta\eta/(\zeta + \eta + 1); \quad \Delta f_{cr} \geq 0.333 \text{ for } f_c > 0 \tag{33}$$

No instability can occur for any value of the system parameters if $\Delta f < 0.333$. Although it is not intuitively clear why 0.333 is the lower bound for Δf_{cr} or why −0.25 is an upper bound for Δg_{cr} and Δh_{cr}, these bounds show the insensitivity of the optimal feedback systems to parameter variations.

V.III. System Instability due to Variation in m

The effect of errors in m on system stability differ from those of the other parameters in the following way. First, the magnitude of the error rather than the relative error influences system performance and thus instability. Secondly, the magnitude of the error which does cause instability is always dependent upon the other values of the system parameters. Note that the value of m does not enter into the calculation of either the filter or feedback gains and thus does not influence optimal system performance. However, system deterioration because of the errors in m can be so severe that instability can occur for stable dynamics ($f_c < 0$) as well as unstable dynamics ($f_c > 0$).

Using Eq. (28) with $f_c > 0$, if Δm is bounded as

$$-\frac{(\zeta + \eta)}{[1 + \zeta][1 + \eta]} < \frac{f_c \Delta m}{h_c g_c} < \frac{\zeta\eta}{[1 + \zeta][1 + \eta]}; \quad f_c > 0 \tag{34}$$

then no system instability will occur. If $f_c\Delta m/h_c g_c$ is less than the lower bound, then an oscillatory divergence occurs; if $f_c\Delta m/h_c g_c$ is above its upper bound, then exponential divergence occurs. The value of Δm for instability is highly dependent upon f_c, h_c, and g_c. Note also that instability occurs for both positive and negative variations in Δm.

If $f_c < 0$, parameter variations in Δm also cause instability. From Eq. (28)

$$-\frac{\Gamma}{f_c} = \begin{bmatrix} -\zeta, & (\Delta m f_c/g_c h_c)(1 - \zeta)(1 - \eta) \\ 1 - \zeta, & -\eta + (\Delta m f_c/g_c h_c)(1 - \zeta)(1 - \eta) \end{bmatrix} \tag{35}$$

If ν is an eigenvalue of Eq. (35), the characteristic equation is

$$\nu^2 + [\zeta + \eta - (\Delta m f_c/g_c h_c)(1 - \zeta)(1 - \eta)]\nu + [\zeta\eta - (\Delta m f_c/g_c h_c)(1 - \zeta)(1 - \eta)] \tag{36}$$

For $\zeta + \eta < \zeta\eta$ the real part of the eigenvalue will be positive if

$$\Delta m f_c/g_c h_c > (\zeta + \eta)/[(1 - \zeta)(1 - \eta)]; \quad f_c < 0 \tag{37}$$

causing an oscillatory divergence. For $\zeta + \eta > \zeta\eta$ the eigenvalue will be positive if

$$\Delta m f_c/g_c h_c > \zeta\eta/[(1 - \zeta)(1 - \eta)]; \quad f_c < 0 \tag{38}$$

causing exponential divergence.

It is obviously harder to induce instability through Δm for stable dynamics ($f_c < 0$) than for unstable dynamics since the denominator $[(1 - \zeta)(1 - \eta)]$ in Eq. (37) and (38) is always smaller than that of Eq. (34). Furthermore, for a given $f_c < 0$, g_c, and h_c, Δm needed to cause instability is always one sign, whereas for $f_c > 0$, positive and negative variation in Δm can cause instability [Eq. (34)].

If the filter and controller were designed separately (as is usually the case in practice), the effect of errors in m will not be detected. However, the effect of errors in m on the total system can be catastrophic. Since only the magnitude of Δm affects system performance, small relative errors in m can produce instability.

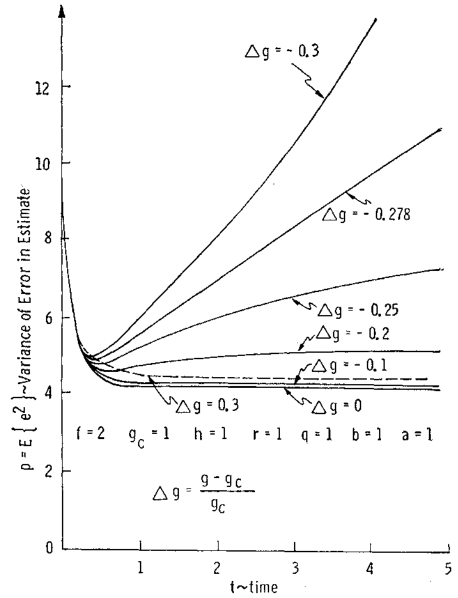

Fig. 2 Variance of error in estimate for model errors in g with $f > 0$.

V.IV. Divergence due to Error in q

If $f_c = 0$ and $q_c = 0$ then the steady-state value of the filter gain is zero. Therefore, Γ of Eq. (28) reduces to

$$\Gamma = \begin{bmatrix} 0 & 0 \\ 0 & -g(a/b)^{1/2} \end{bmatrix} \tag{39}$$

If q is not zero then Ξ of Eq. (22) is

$$\Xi = \begin{bmatrix} q & 0 \\ 0 & 0 \end{bmatrix} \tag{40}$$

The actual covariance of the error in the estimate is propagated as

$$\dot{p} = q \tag{41}$$

Obviously, p diverges when integrating Eq. (41). This is the only situation where instability results due to an error in q. If either q_c or f_c is chosen nonzero (f_c can be either positive or negative) a nonzero filter gain will result. Thus, p will always reach a finite steady-state value because $f - kh$ is negative. This result, obtained by Fitzgerald[1] for filter alone, applies equally well here.

V.V. Results of a Computer Simulation

A computer program was developed to observe the affect of parameter errors on the system transient response. Systems which have acceptable steady state behavior might have an unacceptable transient response.

Since the time interval is still assumed to be infinite, the feedback controller has a constant gain. However, since p_c of Eq. (8) is integrated forward in time its transient response affects the filter gains as

$$k(t) = \frac{f_c}{h_c}\left\{\frac{(\zeta + 1) + (\zeta - 1)Me^{-2f_c\zeta t}}{1 - Me^{-2f_c\zeta t}}\right\} \tag{42}$$

where

$$M = \frac{h_c^2 p_c(0) - f_c r_c(\zeta + 1)}{h_c^2 p_c(0) + f_c r_c(\zeta - 1)} \tag{43}$$

Computer results were obtained for variations of g and m. The 2×2 matrix differential equation of Eq. (20) is numerically integrated in forward time. Although the variance of the estimate and the correlation between the estimate and its error are also calculated only the variance of the error in the estimate is illustrated here.

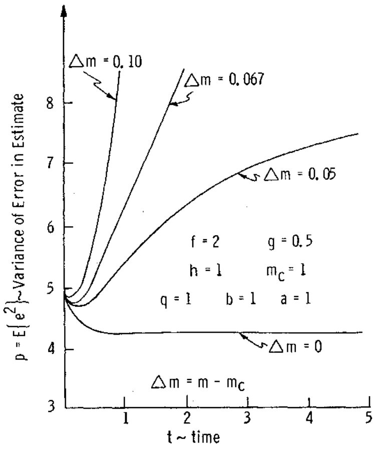

Fig. 3 Variance of the error in the estimate for model errors in m with $f > 0$.

In Fig. 2, the actual value of $p(t)$ is plotted aginst time for various model errors in g. For chosen values of the parameters as

$$f = 2,\ g_c = 1,\ h = 1,\ r = 1,\ q = 1,\quad b = 1,\ a = 1,\ p(0) = 10 \tag{44}$$

the value of Δg to produce a zero eigenvalue is -0.278. Figure 2 shows that a -20% variation of g causes at most a 25% error in $p(t)$. In fact, the system performance is extremely insensitive to model errors in g of less than -10%. However, the system degenerates rapidly as the model error approaches -27.8%. If the controller is designed separately from the filter, an error of $\Delta g = -0.278$ reduces the closed loop feedback pole from -2.736 to -1.09. The control system alone is still quite *stable* but the combined system is unstable. A graph identical to Fig. 2 would be obtained for model errors in Δh if the parameters of Eq. (44) are used.

Positive variation in Δg produced a damped oscillatory behavior. The system behavior degenerates very little even for a Δg of $+30\%$ as shown in the dashed curve in Fig. 2. For the scalar case, the sign of the error is very important in determining system performance.

If model errors in m occur the system performance degenerates very rapidly. In Fig. 3, divergence occurs for $\Delta m = 0.067$ where the system parameters are chosen as

$$f = 1,\ g = 0.5,\ h = 1,\ r = 1,\ q - 1\quad b = 1,\ a = 1,\ p(0) = 5 \tag{45}$$

This error is very small compared to the other system parame-

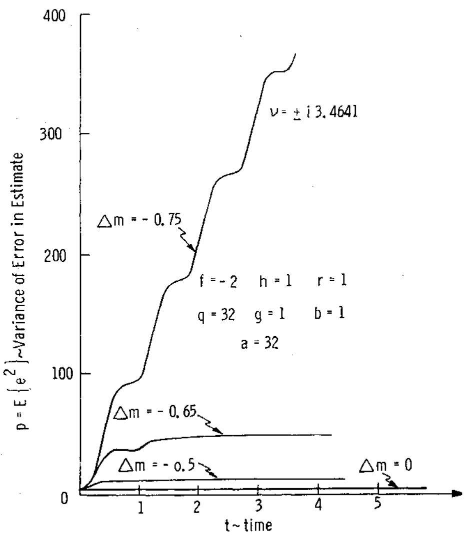

Fig. 4 Variance of the error in the estimate for model errors in m with $f < 0$.

ters. If negative values for f are chosen, Fig. 4 shows the oscillatory but divergent behavior indicated in Sec. V.III. The system parameters are increased to

$$f = -2,\ g = 1,\ h = 1,\ r = 1,\ q = 32,\quad b = 1,\ a = 32,\ p(0) = 5 \tag{46}$$

so that the system gains can be made large enough to cause divergence.

VI. Conclusions and Further Work

This investigation is concerned with the behavior of a system corrupted by model inaccuracies when the filter and controller are designed separately by virtue of the certainty equivalence principle. A general linear $2n \times 2n$ matrix differential equation is determined which propagates the covariance matrix representing the coupled statistics of the estimate of the state and the error in that estimate. If there were no model errors, this differential equation would reduce to two uncoupled $n \times n$ matrix equations.

The study of the effect of model inaccuracies on system performance is begun by considering a one state problem. One main result is that for variations of $\Delta f < 0.333$, and Δg and $\Delta h > -0.25$, respectively, the scalar system always remains stable, independent of all the system parameters. It should be noted that errors in f, h, g, and m have an additive affect on the stability of the system.

The relative errors in f, g, and h are shown to affect system performance. This is in contrast with m for which the magnitude of the error directly affects system performance. This is because neither the optimal filter or controller gains are explicit functions of m. Thus, a small relative error in m may cause system instability.

The effect of errors in r on q generally degrade the steady-state behavior, but do not cause divergence. One important exception is if $f = 0$ and process noise is present where the filter was designed assuming no process noise. For values of $f \neq 0$, instability will not occur because the term $(f - kh) \leq 0$ is zero only when $f = q_c = 0$. This result of Ref. 1 for the filter alone also holds for the filter-controller system.

The matrix case, which should now be investigated, is far more difficult and its results are more subtle. For example, in the scalar case divergence can only occur for negative values of Δg. In the matrix case, an error in G, which amounts to either an increase or decrease in the control gains, may cause instability.

Although a great deal has been done theoretically on stochastic control systems, very few practical applications have been made. Much is to be learned about using linear stochastic controllers in a nonlinear problem. The above indicates a beginning.

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